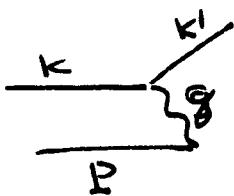


Physics 129a

Solutions to Problem Set #7

$$1. \gamma = \frac{\mathbf{g} \cdot \mathbf{P}}{\mathbf{k} \cdot \mathbf{P}}$$



$$\mathbf{g} = \mathbf{k} - \mathbf{k}'$$

$$\therefore \gamma = \frac{(\mathbf{k} - \mathbf{k}') \cdot \mathbf{P}}{\mathbf{k} \cdot \mathbf{P}} = \frac{\mathbf{k} \cdot \mathbf{P} - \mathbf{k}' \cdot \mathbf{P}}{\mathbf{k} \cdot \mathbf{P}}$$

$$= 1 - \frac{\mathbf{k}' \cdot \mathbf{P}}{\mathbf{k} \cdot \mathbf{P}}$$

$$\Rightarrow \frac{\mathbf{k}' \cdot \mathbf{P}}{\mathbf{k} \cdot \mathbf{P}} = 1 - \gamma$$

The wording of the 2<sup>nd</sup> part is bad.  $\Theta$  is the ~~angle~~ in the quark-electron center-of-mass frame. Then, if energy is high enough that we can ignore masses:

$$\begin{array}{ccc} \xrightarrow{\quad} \mathbf{k}' = (P, P \cos \theta, P \sin \theta, 0) \\ \xleftarrow{\quad \theta} \mathbf{k} = (P, P, 0, 0) \qquad \mathbf{P} = (P, -P, 0, 0) \end{array}$$

$$\downarrow \qquad \qquad \qquad (P, -P \cos \theta, -P \sin \theta, 0)$$

This is because Deep Inelastic Scattering is actually elastic e.g. scattering

$$\frac{\mathbf{k}' \cdot \mathbf{P}}{\mathbf{k} \cdot \mathbf{P}} = \frac{P^2 - P^2 \cos \theta}{P^2 + P^2} = \frac{1 - \cos \theta}{2}$$

2. Perkins eq 5.33 and 5.34 say

$$\frac{F_2^{ep}(x)}{x} = \frac{4}{9} [u + \bar{u}] + \frac{1}{9} [d + \bar{d} + s + \bar{s}]$$

$$\frac{F_2^{en}(x)}{x} = \frac{4}{9} [d + \bar{d}] + \frac{1}{9} [u + \bar{u} + s + \bar{s}]$$

$$\therefore \frac{F_{en}}{F_{ep}} = \frac{\frac{4}{9} [d + \bar{d}] + \frac{1}{9} [u + \bar{u} + s + \bar{s}]}{\frac{4}{9} [u + \bar{u}] + \frac{1}{9} [d + \bar{d} + s + \bar{s}]}$$

The largest value this can have is when all quarks are d quarks

ratio is  $\frac{4/9}{1/9} = 4$

The smallest value is when all quarks are u quarks  
ratio is  $\frac{1/9}{4/9} = 1/4$

Note: if all were s quarks ratio is 1

3. Perkins 5.7 was dropped from the HW (there is a bug in the problem)

4. Perkins 5.8

eq 5.33 says

$$\frac{F_{ep}(x)}{x} = \frac{4}{9} [u(x) + \bar{u}(x)] + \frac{1}{9} [d(x) + \bar{d}(x) + s(x) + \bar{s}(x)]$$

cf 5.34 says

4 | cont

$$\frac{F_2^{en}(x)}{x} = \frac{4}{9} [d(x) + \bar{d}(x)] + \frac{1}{9} [u(x) + \bar{u}(x) + s(x) + \bar{s}(x)]$$

$$\therefore \frac{F_2^{ep}(x) - F_2^{en}(x)}{x} = \frac{1}{3} [u(x) + \bar{u}(x)] + -\frac{1}{3} [d(x) + \bar{d}(x)]$$

Now, write  $u(x) = u_{valence}^{(x)} + u_{seu}^{(x)}$   
and  $d(x) = d_{valence}^{(x)} + d_{seu}^{(x)}$

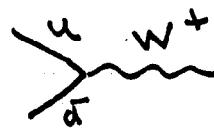
$$\therefore \int (F_2^{ep}(x) - F_2^{en}(x)) \frac{dx}{x} = \int \frac{1}{3} (u_{valence}^{(x)} - d_{valence}^{(x)}) dx + \frac{1}{3} \int u_{seu}^{(x)} + \bar{u}_{...}(x) - d_{seu}^{(x)} - \bar{d}_{...}(x) dx$$

But  $\int_0^1 u_{valence}^{(x)} dx = 2$  and  $\int_0^1 d_{valence}^{(x)} dx = 1$

and  $u_{seu}^{(x)} = \bar{u}(x)$

$$\therefore \int (F_2^{ep}(x) - F_2^{en}(x)) \frac{dx}{x} = \frac{1}{3} (2 - 1) + \frac{2}{3} \int [\bar{u}(x) - \bar{d}(x)] dx \\ = \frac{1}{3} + \frac{2}{3} \int_0^1 [\bar{u}(x) - \bar{d}(x)] dx$$

5. Perkins 6.1



We can relate the total cross section to the cross section for  $q\bar{q}$  annihilation convoluted with the quark distribution functions:

$$\sigma = \int_0^1 \int_0^1 dx_1 dx_2 f_u(x_1) f_{\bar{d}}(x_2) \sigma(x_1 x_2 s)$$

where I have used the fact that  
 $q\bar{q}$  center-of-mass energy =  $x_1 x_2 s$  with  
 $s = p\bar{p}$  center-of-mass energy

So let's start by finding the constants  $a_1 + a_2$ :

We know that the quarks carry  $\frac{1}{2}$  the momentum of the proton so if we ignore sea quarks:

$$\int_0^1 [x u(x) + x d(x)] dx = 0.5$$

$x d(x)$  for proton  $\approx x \bar{d}(x)$  for antiproton

$$\therefore \int_0^1 [a_1 (1-x)^3 + a_2 (1-x)^3] dx = 0.5$$

$$(a_1 + a_2) \int_0^1 (1 - 3x + 3x^2 - x^3) dx = 0.5$$

$$(a_1 + a_2) \left[ x - \frac{3x^2}{2} + \frac{3x^3}{3} - \frac{x^4}{4} \right] \Big|_0^1 = 0.5$$

$$(a_1 + a_2) \left( 1 - \frac{3}{2} + 1 - \frac{1}{4} \right) = 0.5$$

$$(a_1 + a_2) \left( \frac{1}{4} \right) = 0.5 \Rightarrow a_1 + a_2 = 2$$

Also  $\int x u(x) dx = 2 \int x d(x) dx$   
 (ignoring sea quarks)

$$\therefore a_1 = 2a_2 \Rightarrow 2a_2 + a_2 = 2$$

$$\Rightarrow a_2 = 2/3 \quad a_1 = 4/3$$

Now, on to the cross section:

$$\sigma = \int dx_1 dx_2 u(x_1) \bar{d}(x_2) \sigma(x_1 x_2 s)$$

Here  $s = (p\bar{p} \text{ center-of-mass energy})^2$   
 $x_1 x_2 s = (u\bar{d} \text{ center of mass energy})^2$

This problem is easiest if we use the relativistic Breit-Wigner form (see Perkins eq 2.32)

$$\sigma(\hat{s}) = \sigma_{\max} \frac{M_0^2 \pi^2}{(\hat{s} - M_0^2)^2 + \pi^2 M_0^2}$$

Here  $M_0 \equiv M_W$ .

We can write

$$\begin{aligned} \int_{-\infty}^{\infty} \sigma(\hat{s}) d\hat{s} &= \int \frac{1}{(\hat{s} - M_0^2)^2 + M_0^2 \pi^2} d\hat{s} \\ &= \frac{1}{M_0 \pi} \left[ \tan^{-1} \left( \frac{\hat{s} - M_0^2}{M_0 \pi} \right) \right] \Big|_{-\infty}^{\infty} = \frac{\pi}{M_0 \pi} \end{aligned}$$

then using a small width approx:

$$\sigma(\hat{s}) \approx \sigma_{\max} \frac{\pi M_0^2 \delta(\hat{s} - M_0^2)}{s}$$

$$\sigma = \int_0^1 \int_0^1 dx_1 dx_2 u(x_1) \bar{d}(x_2) \sigma(x_1, x_2 s)$$

$$= \int_0^1 \int_0^1 dx_1 dx_2 u(x_1) \bar{d}(x_2) \sigma_{max} \frac{\pi}{m_w r} \delta(x_1 x_2 s - m_w^2)$$

$$= \int_0^1 \int_0^1 dx_1 dx_2 u(x_1) \bar{d}(x_2) \frac{\pi \sigma_{max}}{m_w r} \delta(x_1 - \frac{m_w^2}{x_2 s})$$

$$= \int_P^1 dx_2 u\left(\frac{m_w^2}{x_2 s}\right) \bar{d}(x_2) \frac{\pi}{m_w r} \sigma_{max}$$

where  $P = \frac{m_w^2}{s}$  (since  $x_1 x_2 s = m_w^2$  and  $x_1 \leq 1, x_2 \geq \frac{m_w^2}{s}$ )

$$= \left(\frac{2}{3}\right)\left(\frac{4}{3}\right) \frac{\pi \sigma_{max} m_w r}{s} \int_P^1 \frac{\left(1 - \frac{m_w^2}{x_2}\right)^3}{\frac{m_w^2}{x_2 s}} \frac{(1-x_2)^3}{x_2} \frac{dx_2}{x_2}$$

$$= \frac{8}{9} \pi \frac{\sigma_{max} r}{m_w} \int_P^1 \left(1 - \frac{m_w^2}{x_2}\right)^3 (1-x_2)^3 \frac{dx_2}{x_2}$$